Interaction of Electromagnetic Waves with General Bianisotropic Slabs

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Abstract—**In a number of recent papers an efficient, elegant and systematic formulation technique has been developed which, combining Fourier transform with matrix analysis methods, was found to be quite suitable for problems related to radiation by dipole or other sources in the presence of an arbitrarily general stratified anisotropic medium. In the present paper this technique is adapted and further extended so as to allow the presence of general bianisotropic media described by four tensors with no limitations on their elements. Two specific applications pertaining to some canonical problems of fundamental importance are included to exemplify the method and demonstrate its usefulness. Considered here are: a) radiation by an arbitrarily oriented elementary electric dipole source located in the vicinity of a general bianisotropic slab, either grounded or ungrounded, leading to the expressions of the dyadic Green's function of the structure, and b) reflection and transmission of an arbitrarily polarized plane wave incident upon such a slab, leading to closed-form concise expressions for the reflection and transmission coefficient matrices. These derivations may serve as a basis for formulating and solving numerous propagation, radiation and scattering problems for planar structures which use such materials as substrates.**

I. Introduction

LAYERED anisotropic waveguides and related structures may find potential use in a very broad field of applications; they include integrated-circuit technology for microwave and millimeter wave applications, integrated optics, optoelectronics, nonlinear signal processing, laser beam modulation, optical filtering, geophysical exploration, ionospheric research etc.

The corresponding boundary value problems are characterized, however, by an almost forbidding algebraic complexity when treated in the context of conventional techniques (trying to manipulate a fourth degree rather unwieldy wave equation). An alternative very fruitful formulation, introduced by Berreman [1] and extensively used in recent years by many researchers [2]–[7], is to properly recast Maxwell's equations into a first-order system of coupled differential equations of the form:

$$\frac{d\overline{w}}{dz} = \overline{\overline{P}}\,\overline{w}(z) \qquad (1)$$

where z is the axis of stratification, $\overline{w}$ stands for a vector formed by the $\hat{x}$ and $\hat{y}$ components of the electromagnetic field and $\overline{\overline{P}}$ denotes a z-independent matrix.

In solving these reduced equations by methods of matrix analysis the most important step is that of generating the transition matrix of the system. This task has been carried out in the past mainly by solving a tedious eigenvalue—eigenvector problem, or, alternatively—in the light of these complexities—by using finite-difference algorithms or various infinite series expansions for the transition matrix.

In this context, an extremely powerful, simple and systematic analytical technique has been recently presented in a sequence of papers [8]–[10] for solving the basic equations formulating a variety of problems related to radiation and propagation of electromagnetic waves in the presence of an arbitrarily general layered anisotropic medium. What differentiates this novel approach from other similar ones and leads to significant improvements is mainly the possibility—based on repeated use of the Cayley–Hamilton theorem—to express the transition matrix as a four-term linear combination of the form: $c_0\overline{\overline{I}}_4 + c_1\overline{\overline{P}} + c_2\overline{\overline{P}}^2 + c_3\overline{\overline{P}}^3$ where the expansion coefficients c_j ($j = 0, 1, 2, 3$) assume very simple and highly symmetrical four-term algebraic expressions. Thus our algorithm, in addition to being completely automatic, is so simple and efficient that it requires only the trivial and routine task of evaluating the 4×4 matrix $\overline{\overline{P}}^3$.

In the present paper this technique is adapted and further extended so as to allow the presence of arbitrarily general bianisotropic media. The basic formulation is presented in Section II followed by two specific applications which serve to demonstrate the usefulness of the approach as well as its superiority in comparison with alternative ones used in the past by several other authors. The first application (Section III) concerns with the detailed matrix analysis of the radiation problem of an elementary electric dipole source arbitrarily oriented in the air region above a general, grounded or ungrounded bianisotropic slab. This problem leads essentially to the determination of the dyadic Green's function of the structure; its fundamental importance in formulating several propagation, radiation and scattering problems for planar configurations which use such bianisotropic substrates is obvious. Next, in Section IV, assuming incidence of arbitrarily polarized plane waves upon a general bianisotropic slab (grounded on ungrounded), this technique is used, also, to obtain simple

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closed-form expressions for the reflection and transmission coefficient matrices; these results are, also, basic in formulating several scattering problems for bianisotropic slabs loaded by strips, slots, cylinders etc. Finally, in Section V an extension of the results is attempted to the cases of a) inhomogeneous bianisotropic slabs or b) n-layered bianisotropic media.

In the following analysis the exp $(+j\omega t)$ time dependence, assumed for all field quantities, is suppressed throughout.

II. Basic Formulation

With respect to the structure shown in Fig. 1, regions (0) ($z > 0$; air) and (2) ($z < -d$)—assumed to be isotropic—are characterized by the scalar constants (ϵ_i, μ_i, $k_i = \omega\sqrt{\epsilon_i \mu_i}$) ($i = 0, 2$), respectively. Region (1) ($-d < z < 0$), on the other hand, is taken to be occupied by the most general bianisotropic material characterized by four tensors [11]: $\bar{\bar{\epsilon}}$ (relative dielectric permittivity), $\bar{\bar{\mu}}$ (relative magnetic permeability) and $\bar{\bar{\xi}}$, $\bar{\bar{\eta}}$ (cross coupling tensors). All impressed sources are taken to be located inside region (0). When region (2) is filled with an electrically perfect conductor the configuration of a grounded bianisotropic slab, shown in Fig. 2, results in.

A. Description of the Fields Inside the Bianisotropic Slab

The fields ($\bar{E}$, $\bar{H}$) pertaining to region (1) satisfy the basic field equations [11]:

$$\nabla \times \bar{H} = j\omega\epsilon_0(\bar{\bar{\epsilon}}\bar{E} + \bar{\bar{\xi}}\zeta_0\bar{H}) \tag{2a}$$

$$\nabla \times \bar{E} = -j\omega\mu_0(\bar{\bar{\mu}}\bar{H} + \zeta_0^{-1}\bar{\bar{\eta}}\bar{E}); \qquad \zeta_0 = \sqrt{\mu_0/\epsilon_0}. \tag{2b}$$

In (2):

$$\bar{\bar{q}} = \begin{pmatrix} q_{xx} & q_{xy} & q_{xz} \\ q_{yx} & q_{yy} & q_{yz} \\ q_{zx} & q_{zy} & q_{zz} \end{pmatrix} = \left(\begin{array}{c|c} \bar{\bar{q}}_t & \bar{q}_z \\ \hline \bar{q}^z & q_{zz} \end{array}\right); \tag{3}$$

$$\bar{\bar{q}}_t = \begin{pmatrix} q_{xx} & q_{xy} \\ q_{yx} & q_{yy} \end{pmatrix}, \quad \bar{q}_z = \begin{pmatrix} q_{xz} \\ q_{yz} \end{pmatrix}, \quad \bar{q}^z = (q_{zx} \quad q_{zy}); \tag{4}$$

$q \equiv \epsilon, \mu, \xi, \eta$.

Using the notation:

$$\nabla\times = \begin{pmatrix} 0 & -\partial z & \partial y \\ \partial z & 0 & -\partial x \\ -\partial y & \partial x & 0 \end{pmatrix} \tag{5}$$

and working in the double, with respect to x and y, Fourier transform domain defined by:

$$\mathfrak{F}(k_x, k_y; z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(x, y; z) \cdot \exp(jk_x x + jk_y y)\, dx\, dy \tag{6a}$$

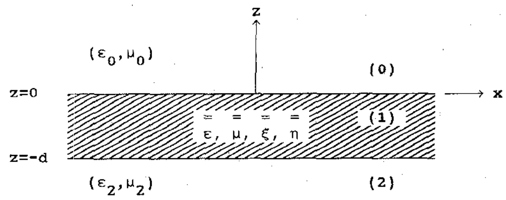

Fig. 1. The geometry of a general bianisotropic slab between two isotropic half-spaces.

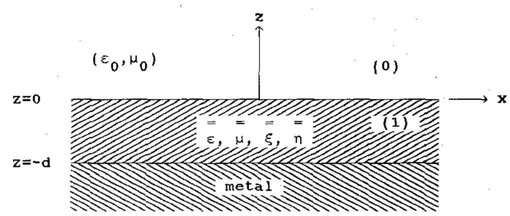

Fig. 2. The geometry of a grounded bianisotropic slab.

$$F(x, y; z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathfrak{F}(k_x, k_y; z) \cdot \exp(-jk_x x - jk_y y)\, dk_x\, dk_y, \tag{6b}$$

$\tilde{\nabla}\times$, the transformed operator of $\nabla\times$ in (5), becomes:

$$\tilde{\nabla}\times = \begin{pmatrix} 0 & -d/dz & -jk_y \\ d/dz & 0 & jk_x \\ jk_y & -jk_x & 0 \end{pmatrix} = \left(\begin{array}{c|c} \bar{\bar{G}}\dfrac{d}{dz} & -j\bar{u} \\ \hline j\bar{u}^T & 0 \end{array}\right). \tag{7}$$

In (7) the shorthand notation:

$$\bar{\bar{G}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -\bar{\bar{G}}^{-1} \tag{8}$$

$$\bar{u} = \begin{pmatrix} k_y \\ -k_x \end{pmatrix}, \quad \bar{u}^T = (k_y \quad -k_x) \tag{9}$$

was used.

Via (3)-(4) and (7)-(9), (2a) yields

$$\bar{\bar{G}}\frac{d}{dz}\bar{\mathcal{H}}_t - j\bar{u}\mathcal{H}_z = jk_0\zeta_0^{-1}(\bar{\bar{\epsilon}}_t\bar{\mathcal{E}}_t + \bar{\epsilon}_z\mathcal{E}_z) + jk_0(\bar{\bar{\xi}}_t\bar{\mathcal{H}}_t + \bar{\xi}_z\mathcal{H}_z) \tag{10a}$$

$$j\bar{u}^T\bar{\mathcal{H}}_t = jk_0\zeta_0^{-1}(\bar{\epsilon}^z\bar{\mathcal{E}}_t + \epsilon_{zz}\mathcal{E}_z) + jk_0(\bar{\xi}^z\bar{\mathcal{H}}_t + \xi_{zz}\mathcal{H}_z). \tag{10b}$$

From (2b), by duality [12] ($\epsilon \leftrightarrow \mu$, $\xi \leftrightarrow -\eta$, $n \leftrightarrow -\xi$, $\mathcal{E} \rightarrow \mathcal{H}$, $\mathcal{H} \rightarrow -\mathcal{E}$), we get

$$-\bar{\bar{G}}\frac{d}{dz}\bar{\mathcal{E}}_t + j\bar{u}\mathcal{E}_z = jk_0\zeta_0(\bar{\bar{\mu}}_t\bar{\mathcal{H}}_t + \bar{\mu}_z\mathcal{H}_z) + jk_0(\bar{\bar{\eta}}_t\bar{\mathcal{E}}_t + \bar{\eta}_z\mathcal{E}_z) \tag{11a}$$

$$-j\bar{u}^T\bar{\mathcal{E}}_t = jk_0\zeta_0(\bar{\mu}^z\bar{\mathcal{H}}_t + \mu_{zz}\mathcal{H}_z) + jk_0(\bar{\eta}^z\bar{\mathcal{E}}_t + \eta_{zz}\mathcal{E}_z). \tag{11b}$$

In (10)–(11):

$$\overline{\mathcal{Q}}_t(z) \equiv \overline{\mathcal{Q}}_t(k_x, k_y; z) = \begin{pmatrix} \mathcal{Q}_x(k_x, k_y; z) \\ \mathcal{Q}_y(k_x, k_y; z) \end{pmatrix},$$

$$\mathcal{Q}_z(z) \equiv \mathcal{Q}_z(k_x, k_y; z); \qquad \mathcal{Q} \equiv \mathcal{E}, \mathcal{H} \tag{12}$$

stand for the Fourier transformed components of the field.

Combining (10b) with (11b) yields

$$\begin{pmatrix} \mathcal{E}_z \\ \mathcal{H}_z \end{pmatrix} = -\frac{1}{D}\begin{pmatrix} \xi_{zz} & \mu_{zz}\zeta_0 \\ -\epsilon_{zz}\zeta_0^{-1} & -\eta_{zz} \end{pmatrix} \cdot \left(\begin{array}{c|c} -\overline{\eta}^z - k_0^{-1}\overline{u}^T & -\zeta_0\overline{\mu}^z \\ \hline \zeta_0^{-1}\overline{\epsilon}^z & \overline{\xi}^z - k_0^{-1}\overline{u}^T \end{array}\right)\begin{pmatrix} \overline{\mathcal{E}}_t \\ \overline{\mathcal{H}}_t \end{pmatrix} \tag{13a}$$

$$D = \epsilon_{zz}\mu_{zz} - \eta_{zz}\xi_{zz}. \tag{13b}$$

Finally, combining (10a) with (11a) and using (13) and (8) leads to the following first-order system of coupled differential equations:

$$\frac{d}{dz}\begin{pmatrix} \overline{\overline{G}}\,\overline{\mathcal{E}}_t \\ \overline{\overline{G}}\,\overline{\mathcal{H}}_t \end{pmatrix} = jk_0\left(\begin{array}{c|c} \overline{\overline{A}}(\epsilon, \mu, \xi, \eta) & \overline{\overline{B}}(\epsilon, \mu, \xi, \eta) \\ \hline \overline{\overline{\Gamma}}(\epsilon, \mu, \xi, \eta) & \overline{\overline{\Delta}}(\epsilon, \mu, \xi, \eta) \end{array}\right)\begin{pmatrix} \overline{\mathcal{E}}_t \\ \overline{\mathcal{H}}_t \end{pmatrix} \tag{14}$$

where, after some simplifications,

$$\overline{\overline{A}}(\epsilon, \mu, \xi, \eta) = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

$$= -\overline{\overline{\eta}}_t - \frac{1}{D}\{(k_0^{-1}\overline{u} - \overline{\eta}_z) \cdot [\xi_{zz}(-\overline{\eta}^z - k_0^{-1}\overline{u}^T) + \mu_{zz}\overline{\epsilon}^z] - \overline{\mu}_z[\epsilon_{zz}(\overline{\eta}^z + k_0^{-1}\overline{u}^T) - \eta_{zz}\overline{\epsilon}^z]\} \tag{15}$$

$$\overline{\overline{B}}(\epsilon, \mu, \xi, \eta) = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

$$= -\zeta_0\overline{\overline{\mu}}_t - \frac{\zeta_0}{D}\{(k_0^{-1}\overline{u} - \overline{\eta}_z) \cdot [\mu_{zz}(\overline{\xi}^z - k_0^{-1}\overline{u}^T) - \xi_{zz}\overline{\mu}^z] - \overline{\mu}_z[-\eta_{zz}(\overline{\xi}^z - k_0^{-1}\overline{u}^T) + \epsilon_{zz}\overline{\mu}^z]\}. \tag{16}$$

The evaluation of the elements of the 2×2 matrices $\overline{\overline{A}}$ and $\overline{\overline{B}}$ from (15) and (16) is quite straightforward. On the other hand, one may readily verify that $\overline{\overline{\Gamma}}$ and $\overline{\overline{\Delta}}$ satisfy the following symmetry (duality) relations:

$$\overline{\overline{\Gamma}}(\epsilon, \mu, \xi, \eta) = -\overline{\overline{B}}(\mu, \epsilon, -\eta, -\xi) \tag{17}$$

$$\overline{\overline{\Delta}}(\epsilon, \mu, \xi, \eta) = \overline{\overline{A}}(\mu, \epsilon, -\eta, -\xi) \tag{18}$$

which greatly add to the simplicity of the final expressions. In view of (15)–(18), (14) is rewritten as

$$\frac{d}{dz}\begin{pmatrix} \overline{\mathcal{E}}_t \\ \overline{\mathcal{H}}_t \end{pmatrix} = -jk_0\left[\begin{array}{cc|cc} -A_{21} & -A_{22} & -B_{21} & -B_{22} \\ A_{11} & A_{12} & B_{11} & B_{12} \\ \hline B_{21}^* & B_{22}^* & -A_{21}^* & -A_{22}^* \\ -B_{11}^* & -B_{12}^* & A_{11}^* & A_{12}^* \end{array}\right] \cdot \begin{pmatrix} \overline{\mathcal{E}}_t \\ \overline{\mathcal{H}}_t \end{pmatrix}$$

$$= \overline{\overline{P}}(k_x, k_y; \epsilon, \mu, \xi, \eta)\begin{pmatrix} \overline{\mathcal{E}}_t \\ \overline{\mathcal{H}}_t \end{pmatrix} \tag{19}$$

where the stared quantities are defined by the duality relations:

$$Q_{ij}^*(\epsilon, \mu, \xi, \eta) = Q_{ij}(\mu, \epsilon, -\eta, -\xi); \qquad i, j = 1, 2; \; Q \equiv A, B. \tag{20}$$

As a partial check of the correctness of (15)–(20), (9)–(11) of [14]-valid for $\overline{\overline{\xi}} = 0$, $\overline{\overline{\eta}} = 0$ (anisotropic medium) and for $k_x = k_0 \sin\theta$, $k_y = 0$-may be derived very simply (taking into account the different time dependence used here).

B. General Solution of (19)

Following [8]–[10] the general solution of (19) may be written as

$$\begin{pmatrix} \overline{\mathcal{E}}_t(z) \\ \overline{\mathcal{H}}_t(z) \end{pmatrix} = \exp(z\overline{\overline{P}})\begin{pmatrix} \overline{\mathcal{E}}_t(0) \\ \overline{\mathcal{H}}_t(0) \end{pmatrix} = \overline{\overline{T}}(k_x, k_y; z)\begin{pmatrix} \overline{\mathcal{E}}_t(0) \\ \overline{\mathcal{H}}_t(0) \end{pmatrix} (-d < z < 0). \tag{21}$$

In (21),

$$\overline{\overline{T}}(z) = \overline{\overline{T}}(k_x, k_y; z) = \begin{pmatrix} \overline{\overline{T}}_1(k_x, k_y; z) & \overline{\overline{T}}_2(k_x, k_y; z) \\ \overline{\overline{T}}_3(k_x, k_y; z) & \overline{\overline{T}}_4(k_x, k_y; z) \end{pmatrix} \equiv \exp(z\overline{\overline{P}}) \tag{22a}$$

($\overline{\overline{T}}_1$, $\overline{\overline{T}}_2$, $\overline{\overline{T}}_3$, $\overline{\overline{T}}_4$ being 2×2 sub-matrices) stands for the 4×4 transition matrix of the differential system (19). As stated in the introduction, this matrix may most simply be expressed in the form of a four-term linear combination of powers of $\overline{\overline{P}}$:

$$\overline{\overline{T}}(z) = c_0(z)\overline{\overline{I}}_4 + c_1(z)\overline{\overline{P}} + c_2(z)\overline{\overline{P}}^2 + c_3(z)\overline{\overline{P}}^3. \tag{22b}$$

The expansion scalar constants c_0, c_1, c_2 and c_3 may be determined in a very elegant manner from the Vandermode-type linear algebraic system:

$$\begin{bmatrix} 1 & \lambda_0 & \lambda_0^2 & \lambda_0^3 \\ 1 & \lambda_1 & \lambda_1^2 & \lambda_1^3 \\ 1 & \lambda_2 & \lambda_2^2 & \lambda_2^3 \\ 1 & \lambda_3 & \lambda_3^2 & \lambda_3^3 \end{bmatrix}\begin{bmatrix} c_0(z) \\ c_1(z) \\ c_2(z) \\ c_3(z) \end{bmatrix} = \begin{bmatrix} \exp(\lambda_0 z) \\ \exp(\lambda_1 z) \\ \exp(\lambda_2 z) \\ \exp(\lambda_3 z) \end{bmatrix}. \tag{23}$$

Following some properties of the Vandermode matrix, these coefficients can be expressed explicitly via the highly symmetrical relations:

$$c_0(z) = -\lambda_1\lambda_2\lambda_3 F_0(z) - \lambda_0\lambda_2\lambda_3 F_1(z) - \lambda_0\lambda_1\lambda_3 F_2(z) - \lambda_0\lambda_1\lambda_2 F_3(z) \quad (24a)$$

$$c_1(z) = (\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)F_0(z) + (\lambda_0\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_0)F_1(z) + (\lambda_0\lambda_1 + \lambda_1\lambda_3 + \lambda_3\lambda_0)F_2(z) + (\lambda_0\lambda_1 + \lambda_1\lambda_2 + \lambda_2\lambda_0)F_3(z) \quad (24b)$$

$$c_2(z) = -(\lambda_1 + \lambda_2 + \lambda_3)F_0(z) - (\lambda_0 + \lambda_2 + \lambda_3)F_1(z) - (\lambda_0 + \lambda_1 + \lambda_3)F_2(z) - (\lambda_0 + \lambda_1 + \lambda_2)F_3(z) \quad (24c)$$

$$c_3(z) = F_0(z) + F_1(z) + F_2(z) + F_3(z) \quad (24d)$$

$$F_0(z) = \frac{\exp(\lambda_0 z)}{(\lambda_0 - \lambda_1)(\lambda_0 - \lambda_2)(\lambda_0 - \lambda_3)} \quad (25a)$$

$$F_1(z) = \frac{\exp(\lambda_1 z)}{(\lambda_1 - \lambda_0)(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \quad (25b)$$

$$F_2(z) = \frac{\exp(\lambda_2 z)}{(\lambda_2 - \lambda_0)(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \quad (25c)$$

$$F_3(z) = \frac{\exp(\lambda_3 z)}{(\lambda_3 - \lambda_0)(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)}. \quad (25d)$$

In (23)–(25) λ_i ($i = 0, 1, 2, 3$)—the four eigenvalues (taken to be distinct) of $P(k_x, k_y)$—are defined by

$$\det(\lambda \bar{I}_4 - \bar{\bar{P}}) \equiv \lambda^4 + \alpha_1\lambda^3 + \alpha_2\lambda^2 + \alpha_3\lambda + \alpha_4 = 0. \quad (26)$$

The coefficients α_i ($i = 1, 2, 3, 4$) of the characteristic equation (26) may, also, be expressed explicitly in terms of $\bar{\bar{P}}$ through the relations:

$$\alpha_1 = -tr(\bar{\bar{P}}) \quad (27a)$$

$$\alpha_2 = -\tfrac{1}{2}[\alpha_1\, tr(\bar{\bar{P}}) + tr(\bar{\bar{P}}^2)] \quad (27b)$$

$$\alpha_3 = -\tfrac{1}{3}[\alpha_2 tr(\bar{\bar{P}}) + \alpha_1 tr(\bar{\bar{P}}^2) + tr(\bar{\bar{P}}^3)] \quad (27c)$$

$$\alpha_4 = \det(\bar{\bar{P}}). \quad (27d)$$

What has been up to now accomplished is a unified general description of the fields inside a general bianisotropic slab through concise, elegant and highly symmetrical closed-form expressions in terms of the 4 × 4 matrices $\bar{\bar{P}}$, $\bar{\bar{P}}^2$ and $\bar{\bar{P}}^3$ solely. Comparing with alternative matrix approaches used in the past, the economy and simplicity of our algorithm are striking. Furthermore, this field representation is quite suited to the application of the boundary conditions as will be seen below.

III. Green's Function of the Structure of a General Bianisotropic Slab

A. Slab Between Two Dielectric Half-Spaces

Consider the elementary, arbitrarily oriented electric dipole source:

$$\bar{J}(\bar{r}) = \bar{p}_e\, \delta(\bar{r} - \bar{r}') \quad (28)$$

having moment $\bar{p}_e$ and located at $\bar{r}' = (x', y', z')$ inside region (0) of the structure shown in Fig. 1. The Fourier transform of the fields excited at points $\bar{r} = (x, y, z)$ of this region may be expressed as a superposition of a primary excitation term (denoted by the superscript "p") added to a complementary one as follows:

$$\begin{pmatrix} \bar{\mathcal{E}}_t(z) \\ \bar{\mathcal{H}}_t(z) \end{pmatrix}^{(0)} = \begin{pmatrix} \bar{\mathcal{E}}_t^p(z) \\ \bar{\mathcal{H}}_t^p(z) \end{pmatrix} + \begin{pmatrix} \bar{\bar{L}}_0 \\ \bar{\bar{M}}_0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} e^{-\gamma_0 z} \quad (29a)$$

where β_1, β_2 denote z-independent expansion constants,

$$\begin{pmatrix} \bar{\mathcal{E}}_t^p(z) \\ \bar{\mathcal{H}}_t^p(z) \end{pmatrix} = \frac{1}{8\pi^2} \left[\begin{matrix} j\zeta_0 k_0^{-1} \begin{pmatrix} k_x^2 - k_0^2 & k_x k_y & -j\gamma_0 s k_x \\ k_x k_y & k_y^2 - k_0^2 & -j\gamma_0 s k_y \end{pmatrix} \\ \begin{matrix} 0 & \gamma_0 s & -jk_y \\ -\gamma_0 s & 0 & jk_x \end{matrix} \end{matrix} \right] \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \cdot \frac{e^{jk_x x' + jk_y y' - \gamma_0 |z - z'|}}{\gamma_0}; \quad s = \operatorname{sgn}(z - z') \quad (29b)$$

$$\bar{\bar{L}}_0 = \bar{\bar{L}}(\gamma_0, k_0, \zeta_0) = \begin{pmatrix} -k_0\zeta_0 k_y & j\gamma_0 k_x \\ k_0\zeta_0 k_x & j\gamma_0 k_y \end{pmatrix} \quad (30)$$

$$\bar{\bar{M}}_0 = \bar{\bar{M}}(\gamma_0, k_0, \zeta_0) = \begin{pmatrix} j\gamma_0 k_x & k_0\zeta_0^{-1} k_y \\ j\gamma_0 k_y & -k_0\zeta_0^{-1} k_x \end{pmatrix} \quad (31)$$

and

$$\gamma_0 = (k_x^2 + k_y^2 - k_0^2)^{1/2} \quad (32)$$

($-\pi/2 < \arg(\gamma_0) \le \pi/2$ in conformity with the radiation condition). Following the notation adopted in (12) the subscript "t" is used in (29) to designate field components parallel to the x-y plane.

Similarly, the fields inside region (2) may be expressed as

$$\begin{pmatrix} \bar{\mathcal{E}}_t(z) \\ \bar{\mathcal{H}}_t(z) \end{pmatrix}^{(2)} = \begin{pmatrix} \bar{\bar{L}}_2 \\ \bar{\bar{M}}_2 \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} e^{\gamma_2 (z + d)} \quad (33a)$$

$$\bar{\bar{L}}_2 = \bar{\bar{L}}(-\gamma_2, k_2, \zeta_2),$$

$$\bar{\bar{M}}_2 = \bar{\bar{M}}(-\gamma_2, k_2, \zeta_2); \quad \zeta_2 = \sqrt{\mu_2/\epsilon_2} \quad (33b)$$

$$\gamma_2 = (k_x^2 + k_y^2 - k_2^2)^{1/2}$$
$$(-\pi/2 < \arg(\gamma_2) \le \pi/2). \quad (33c)$$

The scalar constants (δ_1, δ_2) as well as (β_1, β_2) will be determined via the application of the remaining boundary conditions as follows:

a) at $z = -d$:

$$\begin{pmatrix} \bar{\mathcal{E}}_t(-d) \\ \bar{\mathcal{H}}_t(-d) \end{pmatrix}^{(1)} = \begin{pmatrix} \bar{\mathcal{E}}_t(-d) \\ \bar{\mathcal{H}}_t(-d) \end{pmatrix}^{(2)} \Rightarrow \begin{pmatrix} \bar{\bar{T}}_1(-d) & \bar{\bar{T}}_2(-d) \\ \bar{\bar{T}}_3(-d) & \bar{\bar{T}}_4(-d) \end{pmatrix} \cdot \begin{pmatrix} \bar{\mathcal{E}}_t(0) \\ \bar{\mathcal{H}}_t(0) \end{pmatrix} = \begin{pmatrix} \bar{\bar{L}}_2 \\ \bar{\bar{M}}_2 \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \quad (34)$$

and

b) at $z = 0$:

$$\begin{pmatrix} \bar{\mathcal{E}}_t(0) \\ \bar{\mathcal{H}}_t(0) \end{pmatrix}^{(0)} = \begin{pmatrix} \bar{\mathcal{E}}_t(0) \\ \bar{\mathcal{H}}_t(0) \end{pmatrix}^{(1)} \Rightarrow \begin{pmatrix} \bar{\mathcal{E}}_t(0) \\ \bar{\mathcal{H}}_t(0) \end{pmatrix} = \begin{pmatrix} \bar{\bar{L}}_0 \\ \bar{\bar{M}}_0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \bar{\mathcal{E}}_t^p(0) \\ \bar{\mathcal{H}}_t^p(0) \end{pmatrix}. \quad (35)$$

Combining (34) with (35) and using, also, the property

$$\bar{\bar{T}}(d) = \bar{\bar{T}}^{-1}(-d) \quad (36)$$

of the transition matrix yields

$$\left[\begin{pmatrix} \bar{\bar{T}}_1(d) & \bar{\bar{T}}_2(d) \\ \bar{\bar{T}}_3(d) & \bar{\bar{T}}_4(d) \end{pmatrix} \begin{pmatrix} \bar{\bar{L}}_2 \\ \bar{\bar{M}}_2 \end{pmatrix} \,\middle|\, -\begin{pmatrix} \bar{\bar{L}}_0 \\ \bar{\bar{M}}_0 \end{pmatrix} \right] \begin{bmatrix} \delta_1 \\ \delta_2 \\ \hline \beta_1 \\ \beta_2 \end{bmatrix} = \begin{pmatrix} \bar{\mathcal{E}}_t^p(0) \\ \bar{\mathcal{H}}_t^p(0) \end{pmatrix}. \quad (37)$$

From (37) by inversion and using (29b) one gets

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \frac{1}{8\pi^2} \begin{pmatrix} \bar{\bar{T}}_1(d)\bar{\bar{L}}_2 + \bar{\bar{T}}_2(d)\bar{\bar{M}}_2 & -\bar{\bar{L}}_0 \\ \bar{\bar{T}}_3(d)\bar{\bar{L}}_2 + \bar{\bar{T}}_4(d)\bar{\bar{M}}_2 & -\bar{\bar{M}}_0 \end{pmatrix}^{-1} \cdot \begin{bmatrix} j\zeta_0 k_0^{-1} \begin{pmatrix} k_x^2 - k_0^2 & k_x k_y & j\gamma_0 k_x \\ k_x k_y & k_y^2 - k_0^2 & j\gamma_0 k_y \end{pmatrix} \\ \begin{matrix} 0 & -\gamma_0 & -jk_y \\ \gamma_0 & 0 & jk_x \end{matrix} \end{bmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \frac{e^{jk_x x' + jk_y y' - \gamma_0 z'}}{\gamma_0}. \quad (38)$$

The inversion of the partitioned 4×4 matrix in (38) can be carried out by standard formulas; after this the determination of (δ_1, δ_2) and (β_1, β_2) is straightforward, leading to the expression of the Fourier transformed dyadic Green's function of the structure (through (29), (33a), (35), and (21)). Finally, by applying the double inverse Fourier transform, the space domain Green's function emerges (completed by a suitable delta function term that properly accounts for the singular behavior of the fields at the source position $\bar{r} = \bar{r}'$ as explained in more detail in [9]).

B. Grounded Slab

With respect to the structure shown in Fig. 2, the following relation:

$$\begin{pmatrix} \bar{\bar{T}}_1(-d) & \bar{\bar{T}}_2(-d) \\ \bar{\bar{T}}_3(-d) & \bar{\bar{T}}_4(-d) \end{pmatrix} \begin{pmatrix} \bar{\mathcal{E}}_t(0) \\ \bar{\mathcal{H}}_t(0) \end{pmatrix} = \begin{pmatrix} \bar{0} \\ \bar{\mathcal{H}}_t^{(1)}(-d) \end{pmatrix} \Rightarrow \begin{pmatrix} \bar{\mathcal{E}}_t(0) \\ \bar{\mathcal{H}}_t(0) \end{pmatrix} = \begin{pmatrix} \bar{\bar{T}}_2(d) \\ \bar{\bar{T}}_4(d) \end{pmatrix} \bar{\mathcal{H}}_t^{(1)}(-d), \quad (39)$$

based on the boundary condition $\bar{\mathcal{E}}_t^{(1)}(-d) = 0$, replaces (34) whereas (35) remains unchanged. Combining (39) and (35) yields:

$$\begin{bmatrix} \bar{\bar{T}}_2(d) & -\bar{\bar{L}}_0 \\ \bar{\bar{T}}_4(d) & -\bar{\bar{M}}_0 \end{bmatrix} \begin{bmatrix} \bar{\mathcal{H}}_t^{(1)}(-d) \\ \hline \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \end{bmatrix} = \begin{pmatrix} \bar{\mathcal{E}}_t^p(0) \\ \bar{\mathcal{H}}_t^p(0) \end{pmatrix} \quad (40)$$

from which one finally gets

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = [\bar{\bar{T}}_4(d)\, \bar{\bar{T}}_2^{-1}(d) \bar{\bar{L}}_0 - \bar{\bar{M}}_0]^{-1} \cdot [\bar{\mathcal{H}}_t^p(0) - \bar{\bar{T}}_4(d)\, \bar{\bar{T}}_2^{-1}(d)\, \bar{\mathcal{E}}_t^p(0)] \quad (41a)$$

$$\bar{\mathcal{H}}_t^{(1)}(-d) = \bar{\bar{T}}_2^{-1}(d) \left[\bar{\bar{L}}_0 \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \bar{\mathcal{E}}_t^p(0) \right]. \quad (41b)$$

Equations (41) complete the determination of the fields everywhere through (39), (21) and (29).

C. Radiation Field

The space wave (radiation) far-field of either of the structures shown in Figs. 1 and 2 may be found in closed form through the following simple expressions (based on the stationary phase asymptotic integration technique [13]):

$$E_{\vartheta}^{(i)}(r, \vartheta, \phi) = j\frac{2\pi}{r\sin\vartheta}[k_x \mathcal{E}_x^{(i)}(k_x, k_y; z^i) + k_y \mathcal{E}_y^{(i)}(k_x, k_y; z^i)] \exp(-jk_i r) \quad (42a)$$

$$E_{\phi}^{(i)}(r, \vartheta, \phi) = -j\frac{2\pi}{r}\cot\vartheta[k_y \mathcal{E}_x^{(i)}(k_x, k_y; z^i) - k_x \mathcal{E}_y^{(i)}(k_x, k_y; z^i)] \exp(-jk_i r). \quad (42b)$$

In (42) (r, ϑ, ϕ) are the coordinates (in the system of spherical coordinates centered at $(x, y, z) = (0, 0, 0)$) of the observation point located inside region (i) ($i = 0$ or $i = 2$), whereas

$$z^i = z' + \quad \text{for} \quad i = 0, \qquad \text{or } z^i = -d \quad \text{for} \quad i = 2 \quad (43)$$

$$k_x = k_i \sin\vartheta \cos\phi, \quad k_y = k_i \sin\vartheta \sin\phi. \quad (44)$$

The quantities $\mathcal{E}_q^{(i)}(k_x, k_y; z^i)$ $(q \equiv x, y)$ in (42) are determined through (29), (33a) and (38). Obviously, the case $i = 2$ (observation point inside region (2)) pertains only to the structure of Fig. 1.

IV. Reflection and Transmission of Plane Waves in the Presence of a Bianisotropic Slab

The case of a general anisotropic slab between two isotropic half-spaces, illuminated by an arbitrarily polarized plane wave, has been treated in [10] and led to compact closed-form expressions for the reflection and transmission coefficient matrices. The extension of the analysis to a general bianisotropic slab will be carried out here. Furthermore, the case of a grounded bianisotropic slab will be considered. This latter problem, in the special case of an $\bar{\bar{\epsilon}} - \bar{\bar{\mu}}$ grounded anisotropic slab has been recently treated in [14] by solving an appropriate Ricatti matrix differential equation.

Consider the following plane wave:

$$\bar{E}^{\text{inc}}(\bar{r}) = \bar{E}_0 \exp[-jk_0\hat{k}^{\text{inc}} \cdot \bar{r}] = \bar{E}_0 \exp[-jk_0(x\sin\psi - z\cos\psi)] \quad (45a)$$

$$\bar{H}^{\text{inc}}(\bar{r}) = \bar{H}_0 \exp[-jk_0\hat{k}^{\text{inc}} \cdot \bar{r}] = (\hat{k}^{\text{inc}} \times \bar{E}^{\text{inc}})/\zeta_0 \quad (45b)$$

incident from region (0) along the unit vector $\hat{k}^{\text{inc}} = \hat{x}\sin\psi - \hat{z}\cos\psi$ (where ψ, measured from the $\hat{z}$ axis counterclockwise, denotes the angle of incidence). By decomposing each of the vectors $\bar{E}_0$ and $\bar{H}_0$ into components parallel and perpendicular to the plane of incidence:

$$\bar{E}_0 = -E_{\text{inc}}^{\text{TM}}\hat{y} - E_{\text{inc}}^{\text{TE}}(\hat{k}^{\text{inc}} \times \hat{y}),$$
$$\bar{H}_0 = [-E_{\text{inc}}^{\text{TM}}(\hat{k}^{\text{inc}} \times \hat{y}) + E_{\text{inc}}^{\text{TE}}\hat{y}]/\zeta_0 \quad (46)$$

this plane wave may be expressed as a superposition of TM and TE (to y-axis) waves. Since the excitation is y-independent and the structure is uniform in the y-direction it is concluded that the response is, also, independent of y (i.e., $\partial/\partial y = 0$ everywhere).

A. Reflection by a Grounded Bianisotropic Slab

In the single—with respect to x—Fourier transform domain the excitation field is written as:

$$\begin{Bmatrix} \mathcal{E}^{\text{inc}}(k_x; z) \\ \bar{\mathcal{H}}^{\text{inc}}(k_x; z) \end{Bmatrix} = \begin{Bmatrix} \bar{E}_0 \\ \bar{H}_0 \end{Bmatrix} \exp(jzk_0\cos\psi)\,\delta(k_x - k_0\sin\psi). \quad (47)$$

Denoting by $(\bar{E}^r(\bar{r}), \bar{H}^r(\bar{r}))$ the field reflected into region (0), it may be, also, written as a superposition of TM and TE (to y) waves as follows:

$$\bar{\mathcal{E}}^r(k_x; z) = [-E_0^{\text{TM}}\hat{y} - E_0^{\text{TE}}(j\gamma_0\hat{x} + k_x\hat{z})/k_0]e^{-\gamma_0 z} \quad (48a)$$

$$\bar{\mathcal{H}}^r(k_x; z) = \frac{1}{\zeta_0}[-E_0^{\text{TM}}(j\gamma_0\hat{x} + k_x\hat{z})/k_0 + E_0^{\text{TE}}\hat{y}]e^{-\gamma_0 z} \quad (48b)$$

(working again in the Fourier transform domain). Finally, the Fourier transformed field excited at points inside the bianisotropic slab is still represented by (21) (where $k_y = 0$ as implied by the condition $\partial/\partial y = 0$). Application of the boundary conditions in the way outlined in the preceding Section II.b leads first to the relation:

$$k_x = k_0\sin\psi \quad (49a)$$

and subsequently to the following equation—analogous to (40):

$$\left[\begin{array}{c|c} \bar{\bar{T}}_2(k_0\sin\psi, 0; d) & -\bar{\bar{W}} \\ \bar{\bar{T}}_4(k_0\sin\psi, 0; d) & -\bar{\bar{Q}} \end{array}\right] \left[\begin{array}{c} \bar{\mathcal{H}}_t^{(1)}(-d) \\ \hline \begin{pmatrix} E_0^{\text{TE}} \\ E_0^{\text{TM}} \end{pmatrix} \end{array}\right] = \begin{pmatrix} -\bar{\bar{U}} \\ \bar{\bar{V}} \end{pmatrix} \begin{pmatrix} E_{\text{inc}}^{\text{TE}} \\ E_{\text{inc}}^{\text{TM}} \end{pmatrix} \quad (49b)$$

where in (49) the shorthand notation:

$$\bar{\bar{W}} = \begin{pmatrix} \cos\psi & 0 \\ 0 & -1 \end{pmatrix}, \quad \bar{\bar{U}} = \begin{pmatrix} \cos\psi & 0 \\ 0 & 1 \end{pmatrix},$$
$$\bar{\bar{Q}} = \frac{1}{\zeta_0}\begin{pmatrix} 0 & \cos\psi \\ 1 & 0 \end{pmatrix}, \quad \bar{\bar{V}} = \frac{1}{\zeta_0}\begin{pmatrix} 0 & -\cos\psi \\ 1 & 0 \end{pmatrix} \quad (50)$$

was used.

Eliminating $\bar{\mathcal{H}}_t^{(1)}(-d)$ from (49b) leads to the following relation between $(E_0^{\text{TE}}, E_0^{\text{TM}})$ and $(E_{\text{inc}}^{\text{TE}}, E_{\text{inc}}^{\text{TM}})$:

$$[\bar{\bar{T}}_4(d)\,\bar{\bar{T}}_2^{-1}(d)\bar{\bar{W}} - \bar{\bar{Q}}]\begin{pmatrix} E_0^{\text{TE}} \\ E_0^{\text{TM}} \end{pmatrix} = [\bar{\bar{T}}_4(d)\,\bar{\bar{T}}_2^{-1}(d)\bar{\bar{U}} + \bar{\bar{V}}]\begin{pmatrix} E_{\text{inc}}^{\text{TE}} \\ E_{\text{inc}}^{\text{TM}} \end{pmatrix}. \quad (51)$$

From (51) one readily gets

$$\begin{pmatrix} E_0^{\mathrm{TE}} \\ E_0^{\mathrm{TM}} \end{pmatrix} = \overline{\overline{R}}_m \begin{pmatrix} E_{\mathrm{inc}}^{\mathrm{TE}} \\ E_{\mathrm{inc}}^{\mathrm{TM}} \end{pmatrix} \tag{52a}$$

where

$$\overline{\overline{R}}_m = [\overline{\overline{T}}_4(d)\,\overline{\overline{T}}_2^{-1}(d)\overline{\overline{W}} - \overline{\overline{Q}}]^{-1}[\overline{\overline{T}}_4(d)\,\overline{\overline{T}}_2^{-1}(d)\overline{\overline{U}} + \overline{\overline{V}}] \tag{52b}$$

is the sought reflection coefficient matrix. In (52) (and (51))

$$\overline{\overline{T}}_p(d) \equiv \overline{\overline{T}}_p(k_0 \sin \psi, 0; d) \qquad (p \equiv 2, 4). \tag{52c}$$

B. Reflection and Transmission Through a General Bianisotropic Slab

In addition to (45)–(48), the following relations may be written in this case for the fields transmitted into region (2) and denoted by $(\overline{E}^t(\overline{r}), \overline{H}^t(\overline{r}))$:

$$\overline{\mathcal{E}}^t(k_x; z) = [-E_2^{\mathrm{TM}}\hat{y} - E_2^{\mathrm{TE}}(-j\gamma_2\hat{x} + k_x\hat{z})/k_2]e^{\gamma_2(z+d)} \tag{53a}$$

$$\overline{\mathcal{H}}^t(k_x; z) = \frac{1}{\zeta_2}[-E_2^{\mathrm{TM}}(-j\gamma_2\hat{x} + k_x\hat{z})/k_2 + E_2^{\mathrm{TE}}\hat{y}]e^{\gamma_2(z+d)}. \tag{53b}$$

Application of the continuity conditions at $z = 0, -d$ yields again (49a) and, also, the relations:

$$\begin{pmatrix} \overline{\overline{T}}_1(-d) & \overline{\overline{T}}_2(-d) \\ \overline{\overline{T}}_3(-d) & \overline{\overline{T}}_4(-d) \end{pmatrix} \begin{pmatrix} \overline{\mathcal{E}}_t(0) \\ \overline{\mathcal{H}}_t(0) \end{pmatrix} = \begin{pmatrix} -\overline{\overline{\Theta}} \\ \overline{\overline{\Lambda}} \end{pmatrix} \begin{pmatrix} E_2^{\mathrm{TE}} \\ E_2^{\mathrm{TM}} \end{pmatrix} \tag{54a}$$

$$\begin{pmatrix} \overline{\mathcal{E}}_t(0) \\ \overline{\mathcal{H}}_t(0) \end{pmatrix} = \begin{pmatrix} -\overline{\overline{U}} \\ \overline{\overline{V}} \end{pmatrix} \begin{pmatrix} E_{\mathrm{inc}}^{\mathrm{TE}} \\ E_{\mathrm{inc}}^{\mathrm{TM}} \end{pmatrix} + \begin{pmatrix} \overline{\overline{W}} \\ \overline{\overline{Q}} \end{pmatrix} \begin{pmatrix} E_0^{\mathrm{TE}} \\ E_0^{\mathrm{TM}} \end{pmatrix} \tag{54b}$$

$$\overline{\overline{\Theta}} = \begin{pmatrix} \sqrt{1 - \sin^2 \psi/\epsilon_r} & 0 \\ 0 & 1 \end{pmatrix},$$

$$\overline{\overline{\Lambda}} = \frac{1}{\zeta_2}\begin{pmatrix} 0 & -\sqrt{1 - \sin^2 \psi/\epsilon_r} \\ 1 & 0 \end{pmatrix}; \qquad \epsilon_r = \epsilon_2/\epsilon_0 \tag{54c}$$

with $\overline{\overline{T}}_p(-d)$ $(p \equiv 1, 2, 3, 4)$ defined as in (52c). Combining (54) yields

$$\left(\begin{array}{c|c} \overline{\overline{W}} & \overline{\overline{T}}_1(d)\overline{\overline{\Theta}} - \overline{\overline{T}}_2(d)\overline{\overline{\Lambda}} \\ \hline \overline{\overline{Q}} & \overline{\overline{T}}_3(d)\overline{\overline{\Theta}} - \overline{\overline{T}}_4(d)\overline{\overline{\Lambda}} \end{array}\right) \left[\begin{array}{c} E_0^{\mathrm{TE}} \\ E_0^{\mathrm{TM}} \\ \hline E_2^{\mathrm{TE}} \\ E_2^{\mathrm{TM}} \end{array}\right] = \begin{pmatrix} \overline{\overline{U}} \\ -\overline{\overline{V}} \end{pmatrix} \begin{pmatrix} E_{\mathrm{inc}}^{\mathrm{TE}} \\ E_{\mathrm{inc}}^{\mathrm{TM}} \end{pmatrix}. \tag{55}$$

It may be shown that (55) is the same as (35) of [10] in the case of an anisotropic medium (taking, also, properly into account the differences in notation). From (55) the following relation results in

$$\begin{pmatrix} E_2^{\mathrm{TE}} \\ E_2^{\mathrm{TM}} \end{pmatrix} = \overline{\overline{T}}_m \begin{pmatrix} E_{\mathrm{inc}}^{\mathrm{TE}} \\ E_{\mathrm{inc}}^{\mathrm{TM}} \end{pmatrix}, \tag{56a}$$

where

$$\overline{\overline{T}}_m = [\overline{\overline{W}}^{-1}[\overline{\overline{T}}_1(d)\overline{\overline{\Theta}} - \overline{\overline{T}}_2(d)\overline{\overline{\Lambda}}] - \overline{\overline{Q}}^{-1}[\overline{\overline{T}}_3(d)\overline{\overline{\Theta}} - \overline{\overline{T}}_4(d)\overline{\overline{\Lambda}}]]^{-1}(\overline{\overline{W}}^{-1}\overline{\overline{U}} + \overline{\overline{Q}}^{-1}\overline{\overline{V}}) \tag{56b}$$

is the transmission coefficient matrix. Also,

$$\begin{pmatrix} E_0^{\mathrm{TE}} \\ E_0^{\mathrm{TM}} \end{pmatrix} = \overline{\overline{R}}_m \begin{pmatrix} E_{\mathrm{inc}}^{\mathrm{TE}} \\ E_{\mathrm{inc}}^{\mathrm{TM}} \end{pmatrix}, \tag{57a}$$

where

$$\overline{\overline{R}}_m = \overline{\overline{W}}^{-1}\overline{\overline{U}} - \overline{\overline{W}}^{-1}[\overline{\overline{T}}_1(d)\overline{\overline{\Theta}} - \overline{\overline{T}}_2(d)\overline{\overline{\Lambda}}]\overline{\overline{T}}_m \tag{57b}$$

is the reflection coefficient matrix pertinent to the present structure.

V. Generalizations

A. n-Layered Bianisotropic Medium

Shown in Fig. 3 is an n-layered structure composed of a stratified general bianisotropic medium (regions designated "b_i"; $i = 1, 2, \cdots, n-1$) between two isotropic half-spaces (regions (0) ($z > 0$) and (2) ($z < -d$)). The excitation may be either an arbitrarily polarized plane wave incident from region (0) and described by (45) or the electric dipole source $\overline{J}(\overline{r})$ of (28) located at $\overline{r}'$ inside region (0).

The extension of the preceding analysis to this more general case is quite straightforward. The presence of the layered bianisotropic medium can be taken into account by simply cascading the transition matrices corresponding to the different regions.

In the case of an electric dipole primary excitation the fields inside the isotropic regions (0) and (2) are still described by (29)–(33) in terms of the scalar expansion constants (β_1, β_2) and (δ_1, δ_2) which are again determined by (38) after replacing

$$\overline{\overline{T}}(d) = \begin{pmatrix} \overline{\overline{T}}_1(d) & \overline{\overline{T}}_2(d) \\ \overline{\overline{T}}_3(d) & \overline{\overline{T}}_4(d) \end{pmatrix}$$

by

$$\overline{\overline{T}} = \begin{pmatrix} \overline{\overline{T}}_1 & \overline{\overline{T}}_2 \\ \overline{\overline{T}}_3 & \overline{\overline{T}}_4 \end{pmatrix} = \overline{\overline{T}}^{(1)}(D_1)\,\overline{\overline{T}}^{(2)}(D_2) \cdots \overline{\overline{T}}^{(n-1)}(D_{n-1}). \tag{58a}$$

In (58a):

$$\overline{\overline{T}}^{(i)}(D_i) \equiv \exp[D_i\overline{\overline{P}}^{(i)}]; \quad \overline{\overline{P}}^{(i)} = \overline{\overline{P}}(k_x, k_y; \overline{\overline{\epsilon}}_i, \overline{\overline{\mu}}_i, \overline{\overline{\xi}}_i, \overline{\overline{\eta}}_i) \tag{58b}$$

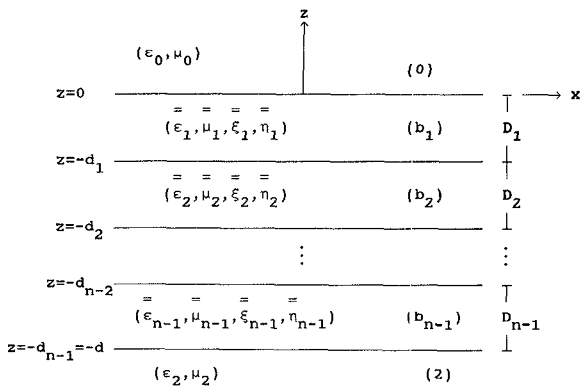

Fig. 3. The geometry of a stratified bianisotropic medium.

is given by (22b) for $\overline{\overline{P}} \equiv \overline{\overline{P}}^{(i)}$ $(i = 1, 2, \cdots, n - 1)$ and corresponds to the transition matrix of region (b_i). In this way and after completing the determination of the field inside regions (0) and (2), the field inside any of the bianisotropic layers (b_i) may be found by either of the following relations:

$$\begin{pmatrix} \overline{\mathcal{E}}_t(z) \\ \overline{\mathcal{H}}_t(z) \end{pmatrix}^{(i)} = \overline{\overline{T}}^{(i)}(z + d_i)\, \overline{\overline{T}}^{(i+1)}(D_{i+1})\, \overline{\overline{T}}^{(i+2)}(D_{i+2}) \cdots \overline{\overline{T}}^{(n-1)}(D_{n-1}) \begin{pmatrix} \overline{\overline{L}}_2 \\ \overline{\overline{M}}_2 \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \quad (59a)$$

or

$$\begin{pmatrix} \overline{\mathcal{E}}_t(z) \\ \overline{\mathcal{H}}_t(z) \end{pmatrix}^{(i)} = \overline{\overline{T}}^{(i)}(z + d_{i-1})\, \overline{\overline{T}}^{(i-1)}(-D_{i-1}) \cdot \overline{\overline{T}}^{(i-2)}(-D_{i-2}) \cdots \overline{\overline{T}}^{(1)}(-D_1) \cdot \left[\begin{pmatrix} \overline{\overline{L}}_0 \\ \overline{\overline{M}}_0 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \overline{\mathcal{E}}_t^p(0) \\ \overline{\mathcal{H}}_t^p(0) \end{pmatrix} \right]. \quad (59b)$$

In the case where the structure is grounded (i.e., when region (2) is filled with a perfect electric conductor) the field is most conveniently given by (59b) after evaluating (β_1, β_2) by (41a) (in terms of $\overline{\overline{T}}_2$ and $\overline{\overline{T}}_4$ given by (58) in place of $\overline{\overline{T}}_2(d)$ and $\overline{\overline{T}}_4(d)$).

In the case of a plane wave excitation, the expressions of the reflection and transmission coefficient matrices derived in the preceding section remain unchanged. Now the block sub-matrices $\overline{\overline{T}}_i$ $(i = 1\text{–}4)$ are determined from (58) for $k_y = 0$, $k_x = k_0 \sin \psi$.

B. Inhomogeneous Bianisotropic Slabs

With respect to the structures of Figs. 1 and 2 we assume now that

$$\overline{\overline{q}} = \overline{\overline{q}}(z); \quad q \equiv \epsilon, \mu, \xi, \eta \qquad (-d < z < 0). \quad (60)$$

An approximate solution can be found in this case by artificially dividing the whole region $(-d < z < 0)$ into $n - 1$ sublayers and using the model of Fig. 3. For sufficiently large n (more precisely, for sufficiently small D_i; $i = 1, 2, \cdots, n - 1$) one may assume a constant value for each of the tensor elements q_{rs} $(q \equiv \epsilon, \mu, \xi, \eta;\ r, s \equiv x, y, z)$ inside each sublayer. The sought approximate solution can then be derived on the basis of the results of the preceding subsection. Alternatively, the transition matrix $\overline{\overline{T}}$ (whose knowledge suffices to complete the solution everywhere) can be found from (58a) after evaluating $\overline{\overline{T}}^{(i)}(D_i)$ $(i = 1, 2, \cdots, n - 1)$ using a Runge-Kutta formula [15; (36)–(37)] correct to within the third power of D_i.

Conclusion

An analytical formulation technique has been presented suitable for treating problems of interaction of electromagnetic waves with the most general bianisotropic layered media. In comparison with alternative matrix methods used in the past by other authors the economy and simplicity of this algorithm are striking. The specific applications considered in this paper lead to the expressions of the dyadic Green's functions for a grounded or ungrounded arbitrarily general bianisotropic slab and, also, to closed-form expressions for the reflection and transmission coefficient matrices. These results may serve as a basis for formulating numerous radiation, scattering and propagation problems for structures loaded by such bianisotropic media. Furthermore, some useful extensions to either inhomogeneous or n-layered bianisotropic structures were considered.

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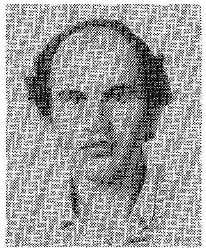

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